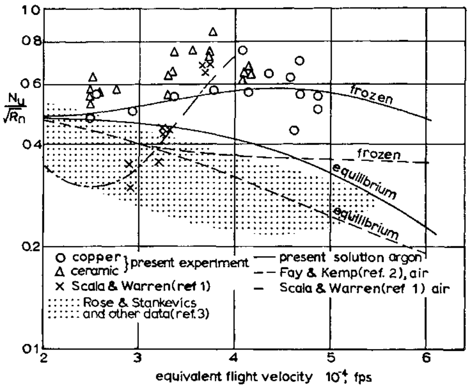

Fig 4 Comparison of heat-transfer results with other data; stagnation-point heat transfer (N_u = Nusselt number, R_n = Reynolds number)

program that is an improved version of the one used in Ref 12 which solves the flow upstream of the throat as well as downstream Significant ionic recombination was predicted by this method The enthalpy levels calculated using the two methods just described agree within the accuracy of measurement (±25%) over the whole operating range

The formula in Ref 13 was used to calculate the local heat-transfer rate over the hemisphere The resulting theoretical heat-transfer rates are shown in Fig 3 As seen from the figure, there is no difference between the heat transfer to the copper and ceramic surface The heat transfer to the cone shows good agreement between theory and experiment The heat-transfer rate to the hemisphere shows that the boundary-layer flow at the stagnation region is nearly frozen The examination of the flow Damköhler number in the boundary layer shows that the flow should be ionically frozen except for the immediate vicinity of the wall Thus, the theory and experiment both support the conclusion that the boundary-layer flow under these conditions is nearly frozen

In order to compare the present result with equivalent data for air, the heat-transfer rate to the hemisphere is plotted in Fig 4 with the equivalent flight speed as abscissa The equivalent flight speed was calculated to give the same enthalpy fraction, i e , the ratio of energy consumed in ionizing to the total energy, as in flight The difference between the experimental results presented here and those of Rose and Stankevics and other similar ones summarized in Ref 3 can be attributed to the difference in test conditions In Ref 3 the flow was in equilibrium; here the flow is nearly frozen The test conditions relevant to Ref 1 are unknown, but from the foregoing it seems that the flow must have been frozen The disagreement between the results of Scala and Warren[1] and the theory of Ref 2 for frozen flow remains unresolved Figure 4 also shows that the difference between the theoretical heat transfer from a frozen and equilibrium boundary layer is more severe for argon than for air This may be because the electron-atom collision cross section in argon is smaller than in air, and hence the frozen Prandtl number decreases more rapidly as the ionization fraction increases than in air

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Effective Heat of Ablation of Graphite

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THE effective heat of ablation, designated Q^* or H_{eff}, is an artifice often used for the prediction or comparison of the performance of ablation materials It is usually defined as[1]

$$Q^* = q_0/\dot{m} \tag{1}$$

where $\dot{m}$ is the mass ablation rate and q_0 is the heat-transfer rate to a nonablating surface which is at the ablation temperature

$$q_0 = (g\rho_e u\, c_{f\,0}/2)(h_{se} - h_{ew}) \tag{2}$$

where ρ_e, u , and h_{se} are, respectively, the density, velocity, and total enthalpy at the edge of the boundary layer; h_{ew} is the enthalpy of the gas that exists at the edge of the boundary layer (air or other medium) evaluated at surface conditions; and $c_{f\,0}$ is the skin-friction coefficient in the absence of ablation With this definition, Q^* is the energy absorbed or blocked per unit mass of material ablated Hence, Q^* is usually considered as a measure of the effectiveness of the ablation material

Graphite does not go through a melting phase, and sublimation does not occur until very high surface temperatures are attained At lower temperatures, ablation of graphite in air occurs because of chemical reaction with oxygen Combustion rates are relatively low, however, so that the heat blockage as a result of mass addition to the boundary layer is minor The heat that is conducted to or radiated from the graphite can actually be increased by the combined effects of the exothermic chemical reaction and the mass transfer, whereas it is markedly reduced for most ablation materials such as the reinforced plastics, fused silica and teflon. Because of the low ablation rates, the effective heat of ablation of graphite is much higher than that of most other ablators, considerable heat being stored in the solid phase and reradiated from the surface There has been considerable misunderstanding regarding the heat of ablation of graphite The purpose of this note is to attempt to clear up some of this confusion and to bring to attention a more convenient method for the rapid estimation of graphite ablation rates [2]

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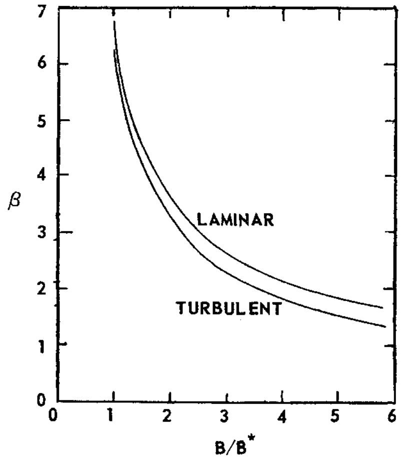

Fig 1 The parameter β for graphite ablation

With Lewis and Prandtl numbers of unity, and in the absence of a pressure-gradient effect, Denison has shown that the ablation rate of a combustible sublimer such as graphite can be expressed as[3 4]

$$\dot{m} = Bg\rho_e u_e c_{fe}/2 \tag{3}$$

where c_f is the skin-friction coefficient including the effects of ablation The parameter B, often called the blowing parameter, is a dimensionless driving force for mass transfer The parameter B can be determined for a given ablator and environmental gas in terms of the thermodynamic state of the gas mixture at the surface, e g , surface temperature and static pressure

In the case of graphite, at temperatures above 2000°F or so, diffusion-controlled surface combustion occurs with CO gas as the primary product of combustion In this event, B is a constant (0 1734) designated B^* At lower temperatures, surface kinetics predominate, and the presence of CO_2 can be important[5] Either of these phenomena will cause B to be less than B^* At very high surface temperatures the graphite begins to sublime, and the combustion reaction is removed from the surface into the boundary layer When this occurs, B exceeds B^* The present note is concerned with diffusion-controlled combustion with and without sublimation The low-temperature regime is discussed at length by Scala[5]

From the preceding equations, Q^* for graphite can be expressed as

$$\begin{aligned} Q^* &= (h_{se} - h_{\ w})/B(c_{fe}/c_{fe0}) \\ &= \beta(h_{se} - h_{ew}) \end{aligned} \tag{4}$$

where $\beta = (Bc_f / c_{f\ 0})^{-1}$ The parameter β can be determined from mass-transfer analyses or experiments in terms of B or B/B^* It is presented for the ablation of graphite in laminar and turbulent boundary layers as a function of B/B^* in Fig 1 The effective heat of ablation is presented in terms of $(h_{se} - h_{\ w})$ for various B/B^* in Fig 2 The results of Scala[5] for stagnation-point ablation with diffusion-controlled surface combustion ($B/B^* = 1$) are also presented in Fig 2 for comparison The agreement is reasonably good, lending confidence to the use of the simplified model of Denison † The present results and those of Scala both indicate that the Q^* for diffusion-controlled surface combustion is an order of magnitude larger than that for most ablators such as silica and phenolic-nylon[1]

It is important to note that as gas phase combustion occurs ($B/B^* > 1$) the Q^* is drastically reduced At first this might seem to be a paradox With gas-phase combustion, the

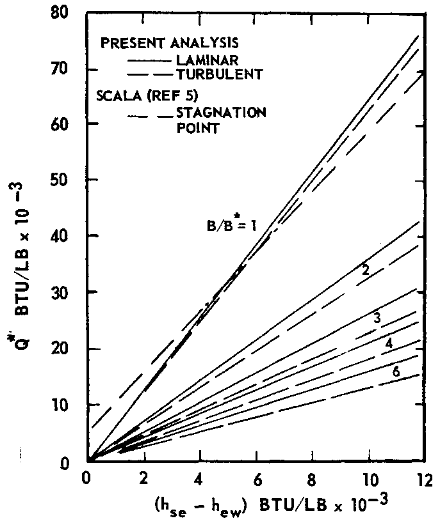

Fig 2 The effective heat of ablation of graphite

graphite becomes a much more efficient ablator as the highly endothermic latent heat of vaporization is utilized; yet, the Q^*, usually considered a measure of the effectiveness of an ablator, is reduced This can be explained as follows When diffusion-controlled surface combustion occurs, the graphite does not behave as a true ablator Rather, it is a radiating heat sink which happens to undergo some surface combustion With very high heat rates, such that significant gas-phase combustion occurs, considerable heat is absorbed in the phase change, and graphite behaves as an ablator as well as a heat sink and a radiator Furthermore, the combustion reaction is removed from the surface so that the energy released to the surface is reduced Under these conditions, the Q^* is still several times that of other ablators

To further illustrate it can be shown[2] that the Q^* for graphite can also be expressed as

$$Q^* = \frac{\lambda_w - (\alpha_w \Gamma_{Oe} Q_{pOw}/B) + \eta(h_{se} - h_{ew})}{1 - (q_{\ ad} + q_{\text{cond}})/q_0} \tag{5}$$

where Q_{pOw} is the heat of combustion for gaseous reactants at surface conditions; α_w is the fraction of the heat of combus-

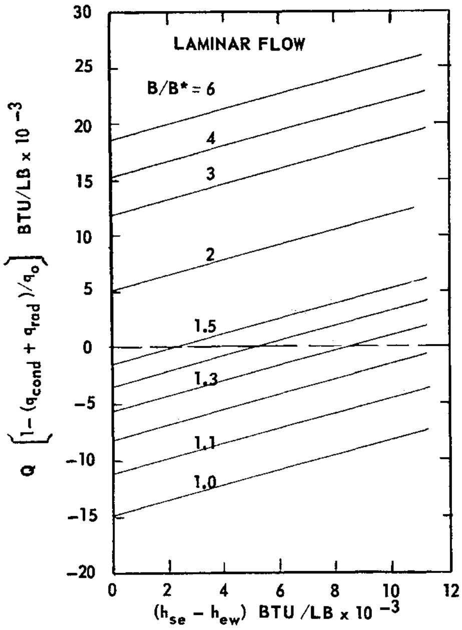

Fig 3 The portion of the effective heat of ablation due to phase change, chemical reaction, and mass addition to the boundary layer

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† Scala has considered the transport properties of the mixture in detail and has included the effects of a pressure gradient and variable Lewis and Prandtl numbers

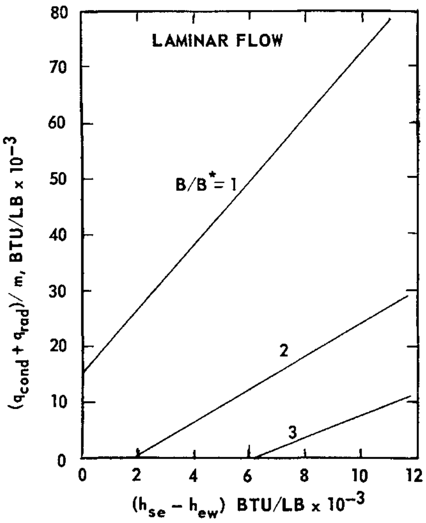

Fig 4 The portion of the effective heat of ablation due to heat conduction and radiation

tion actually produced (usually equal to unity for diffusion-controlled graphite combustion); Γ_{O_e} is the mass fraction of oxygen at the outer edge of the boundary layer; λ_w is the heat of vaporization at surface conditions; q_{cond} is the heat conducted into the graphite; $q_{\ \ d}$ is the heat radiated from the graphite surface; and η is the blowing effectiveness parameter

$$\eta = (1 - c_{f_e}/c_{f_{e0}})/B(c_f\ /c_{f\ 0}) \tag{6}$$

For graphite ablation, η is 0 68 for laminar flow and 0 39 for turbulent flow From Ref 3, λ_w = 25,300 Btu/lb, Q_{pO_w} = 30,200 Btu/lb, and Γ_O = 0 23144

The parameter $Q^*[1 - (q_{\ ad} + q_{cond})/q_0]$ is that portion of Q^* which can be attributed to the ablation process This is presented in Fig 3 for laminar flow as a function of $(h_s - h_{ew})$ with B/B^* as a parameter It can be seen that this quantity is negative for B/B^* less than 1 5 or so With this the case, the net effect of phase change, chemical reaction, and mass addition to the boundary layer is to increase the heat transfer that must be absorbed by the conduction and radiation processes With B/B^* of 2 to 3, the contribution of ablation to Q^* is 10,000 to 20,000 Btu/lb, considerably higher than that of other ablators

The contribution of conduction and radiation to the effective heat of ablation, $(q_{c\ nd} + q_{\ ad})/\dot{m}$, is presented in Fig 4 This is very important for $B/B^* = 1$, especially at high enthalpy levels However, it approaches zero at B/B^* of 2 to 4 dependent upon the enthalpy level, an upper limit in the magnitude of B/B^* for heat transfer to the surface Correspondingly, the maximum possible Q^* for graphite due to the ablation process alone varies from zero for negligible $(h\ \ - h_{ew})$ to nearly 20,000 Btu/lb at $(h_{\ e} - h_{\ w})$ of 10,000 Btu/lb

In order to determine Q^* from Eq (4) or Fig 2, it is necessary to know the magnitude of the blowing parameter B When diffusion-controlled surface combustion occurs, this presents no problem as $B = B^*$ This is the case for surface temperatures greater than 2000°F or so but below 4000° to 6400°F for pressures of 10^{-4} to 100 atm, respectively [2] When heat-transfer rates are sufficiently high to produce higher surface temperatures, gas-phase combustion occurs, and B exceeds B^* The ratio B/B^* is presented in Ref 2 as a function of surface temperature and pressure

Since B must be computed anyway, there seems to be little advantage to the use of Q^* for the rapid estimation of graphite ablation rates since they can be computed directly from Eq (3).

Calculations have been performed for the diffusion-controlled combustion and sublimation regimes over a wide range of flight conditions The results are presented in graphical form in Ref 2 These should be useful for the rapid estimation of graphite ablation rates and surface temperatures for radiation equilibrium or transient heat conduction (thin-skin approximation) The results are presented in terms of the local static pressure, the stagnation enthalpy, and the heat flux to a cold wall They apply to any position on the body and to turbulent as well as laminar flow conditions

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Velocity Defect Law for a Transpired Turbulent Boundary Layer

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A RECENT paper[1] has proposed that a friction velocity based upon the maximum shear stress should be employed in the formulation of the velocity defect law appropriate to a transpired turbulent boundary layer The theoretical calculations reported here have extended Clauser's[2] concepts to this case but are restricted to $dp/dx = 0$ The results support this view

It is assumed that the stream function is described by

$$\psi = U_1\tilde{\delta}f(\eta) - \int V_0(x)dx \tag{1}$$

where $\tilde{\delta}$ is an arbitrary length scale and η is $y/\tilde{\delta}$ This stream function then allows the momentum equation to be written as

$$-f'f + \frac{V_0}{U_1\tilde{\delta}'}f'' = \frac{\epsilon}{\rho U_1\tilde{\delta}\tilde{\delta}'}f''' \tag{2}$$

where ϵ is the eddy viscosity (assumed independent of y) and the primes indicate differentiation with respect to either η or x, whichever is appropriate Since $\tilde{\delta}$ is as yet arbitrary, let it be defined by

$$\frac{V_0}{U_1\tilde{\delta}'} = \frac{\epsilon}{\rho U_1\tilde{\delta}\tilde{\delta}'} = \frac{1}{\beta} = \text{const} \tag{3}$$

The boundary conditions are

$$f'(0) = U(0)/U_1 \qquad f(0) = 0 \qquad f'(\infty) \rightarrow 1 \tag{4}$$

The first condition reflects the fact that this analysis is valid only over the outer portion of the boundary layer, and the velocity profile for this region extrapolates to a finite value

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